

RESEARCH ARTICLE

A New Class of Compounded Distribution based on Lindley - Binomial and Gamma - Geometric Models

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ABSTRACT: In this paper, a new compounded distribution obtained by combining the Lindley Binomial and Gamma Geometric distributions is proposed. Several important statistical properties of the proposed distribution, including the probability mass function, cumulative distribution function, and moment generating function, are derived. Expressions for the mean and variance are also obtained. Reliability characteristics such as the survival function and hazard rate function are investigated. Parameter estimation is discussed using several classical estimation methods including maximum likelihood estimation, method of moments, least squares estimation, Cramer von Mises estimation, and Anderson Darling estimation. A Monte Carlo simulation study is conducted to examine the performance of the estimators in terms of bias and mean squared error. The results indicate that the proposed distribution provides a flexible model for analyzing count data and reliability phenomena.

Keywords: Lindley Binomial Distribution; Gamma Geometric Distribution; Compounded Distribution; Parameter Estimation; Monte Carlo Simulation.

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1. INTRODUCTION

Probability distributions play a fundamental role in modeling random phenomena in various areas such as reliability theory, actuarial science, survival analysis, demography, population studies, and biological sciences. In many practical situations, classical probability distributions are not sufficiently flexible to capture the variability, skewness, dispersion, and tail behaviour observed in real-world data. Consequently, the development of new probability models through transformation,

compounding, and mixture mechanisms has become an important area of research in probability and statistics. Such approaches provide additional parameters and structural flexibility, allowing probability models to accommodate a wider range of empirical characteristics[1]. The development of flexible Lindley-type distributions has received considerable attention because of the simple mathematical structure and useful reliability properties of the Lindley distribution. Zakerzadeh and Mahmoudi [2] introduced a new two-parameter lifetime distribution and investigated its important statistical properties, parameter estimation, and applications to lifetime data. Alzaatreh, Lee, and Famoye [3] proposed a general method for generating families of continuous distributions through the T-X family approach, which subsequently provided a useful framework for constructing flexible extensions of several classical distributions. Jones [4] discussed families of distributions involving additional shape parameters and emphasized the importance of such parameters in controlling skewness and tail behavior.

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Subsequently, several generalized and transformed Lindley-type distributions have been proposed. Shukla [5] investigated a three-parameter generalized Lindley distribution and studied its application in lifetime data analysis. Tahir et al. [6] developed the inverted Nadarajah–Haghighi distribution and investigated different estimation methods and applications, demonstrating the usefulness of transformation-based approaches for constructing flexible lifetime models. Hassan et al. [7] introduced the alpha power transformed extended exponential distribution and investigated its properties, estimation methods, simulation performance, and applications. Elbatal and Khalil [8] proposed a new extension of the Lindley–geometric distribution with enhanced flexibility in the hazard-rate function and demonstrated its applicability through simulation and real-data analysis.

In recent years, particular attention has been given to Lindley-type models for count data. Tharshan and Wijekoon [9] proposed a Poisson modification of the quasi-Lindley distribution specifically for modeling overdispersed count data. The proposed model was further extended through zero modification to accommodate datasets containing an excess number of zeros. This development is particularly relevant in applications where the variance exceeds the mean and where zero observations occur more frequently than expected under standard count distributions. Ngom et al. [10] proposed the pseudo-Lindley alpha power transformed distribution and studied its mathematical, reliability, and asymptotic properties, thereby further extending the flexibility of the Lindley family.

More recently, research has also focused on statistical inference and goodness-of-fit procedures for Lindley-type models. Noughabi and Noughabi [11] developed an empirical likelihood ratio-based goodness-of-fit test for the Lindley distribution and investigated its performance using Monte Carlo simulations. Qayoom et al. [12] proposed a new class of Lindley distribution and studied its reliability properties, estimation procedures, simulation performance, and applications. Their study considered several estimation methods, including maximum likelihood, least squares, weighted least squares, Cramer–von Mises, and Anderson–Darling methods.

The recent literature has continued to expand the Lindley family through inverse transformations, generalized structures, censoring schemes, and goodness-of-fit procedures. Hassan et al. [13] introduced the inverse power XLindley distribution and investigated its statistical inference and applications to engineering data using several estimation procedures. Ren and Shi [14] developed

a generalized Lindley distribution and studied its properties, estimation methods, and applications under progressively Type-II censored samples. Bayesian and classical estimation procedures were considered along with simulation studies and real-data applications. Njuki and Avallone [15] proposed an energy-statistic-based goodness-of-fit test for the Lindley distribution and demonstrated its applicability to lifetime data. Yaghoubi, Khorram, and Arjmand [16] most recently proposed a recursive exponential-gamma mixture as a new generalized Lindley distribution, further demonstrating the usefulness of mixture mechanisms for constructing flexible Lindley-type probability models.

The above literature indicates that the development of Lindley-type distributions has progressed from classical lifetime models to generalized, transformed, inverse, mixture, zero-modified, and count-data models. However, most existing developments have focused either on lifetime and reliability data or on specific count data characteristics such as over-dispersion or zero modification. There remains scope for developing a flexible compounded probability distribution that can exploit the complementary characteristics of different discrete and continuous probability components and provide an appropriate framework for modeling count data with varying dispersion and tail behaviour. This motivates the development of a new compounded distribution based on the Lindley–Binomial and Gamma–Geometric distributions.

Motivated by these developments, the present paper introduces a new compounded distribution obtained by combining the Lindley–Binomial and Gamma–Geometric distributions through a convolution mechanism. The proposed model provides a flexible framework for modeling count data and reliability-related phenomena and is expected to accommodate a wider range of dispersion and tail characteristics than the corresponding component distributions. The compounding mechanism combines the structural features of the two component distributions and consequently provides additional flexibility for statistical modeling.

Several important statistical properties of the proposed distribution are derived. These include the probability mass function (PMF), cumulative distribution function (CDF), and moment generating function (MGF). Important distributional characteristics, such as the mean and variance, are also obtained. In addition, reliability characteristics, including the survival function and hazard rate function, are investigated to examine the behaviour of the proposed model from a reliability perspective.

For parameter estimation, several classical estimation methods are considered, including

maximum likelihood estimation (MLE), method of moments (MOM), least squares estimation (LSE), Cramer–von Mises' estimation (CVM), and Anderson–Darling estimation (AD). A comprehensive simulation study is conducted to investigate the finite-sample performance of these estimators in terms of bias and mean squared error (MSE) under different parameter settings and sample sizes. The practical applicability of the proposed distribution is further examined using appropriate real datasets, and its performance is compared with competing probability models using suitable goodness-of-fit criteria.

Theorem 1.1 : Probability Mass Function

Let X_1 and X_2 be two independent random variables such that

$$X_1 \sim LB(n, \lambda), \quad X_2 \sim GG(\alpha, \beta).$$

Define a new random variable

$$X = X_1 + X_2.$$

Then the probability mass function (PMF) of X is given by

$$P(X = x) = \sum_{k=0}^x P_{LB}(k) P_{GG}(x-k), \quad x = 0, 1, 2, \dots$$

Substituting the component PMFs yields

$$P(X = x) = \sum_{k=0}^x \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} I_{k,x},$$

where

$$I_{k,x} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+x-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

This represents the closed-form integral expression of the PMF of the proposed compounded distribution.

Proof: Since $X = X_1 + X_2$ and X_1 and X_2 are independent random variables, the probability mass function of X can be obtained using the convolution property of discrete random variables:

$$P(X = x) = P(X_1 + X_2 = x).$$

Thus

$$P(X = x) = \sum_{k=0}^x P(X_1 = k) P(X_2 = x - k).$$

Let

$$P_{LB}(k) = P(X_1 = k)$$

be the PMF of the Lindley--Binomial distribution and

$$P_{GG}(y) = P(X_2 = y)$$

be the PMF of the Gamma--Geometric distribution. Substituting these expressions into the convolution formula gives

$$P(X = x) = \sum_{k=0}^x P_{LB}(k) P_{GG}(x - k).$$

Using the explicit forms of $P_{LB}(k)$ and $P_{GG}(y)$, we obtain

$$P(X = x) = \sum_{k=0}^x \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} I_{k,x},$$

where

$$I_{k,x} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+x-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

Hence the required PMF of the proposed compounded distribution is obtained.

Corollary 1.2.: Let

$$C = \frac{\lambda^2 \beta^\alpha}{(1+\lambda) \Gamma(\alpha)}.$$

Then the PMF of the proposed distribution can be written in simplified form as

$$P(X = x) = C \sum_{k=0}^x \binom{n}{k} I_{k,x},$$

where

$$I_{k,x} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+x-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

Proposition 1.3 : The continuous mixing density obtained from the Lindley and Gamma components is given by

$$f(\theta) = \frac{\lambda^2 \beta^\alpha}{(1+\lambda)\Gamma(\alpha)} (1+\theta) \theta^{\alpha-1} e^{-(\lambda+\beta)\theta}, \quad \theta > 0.$$

Theorem 1.4 : Cumulative Distribution Function

Let X_1 and X_2 be two independent random variables such that

$$X_1 \sim LB(n, \lambda), \quad X_2 \sim GG(\alpha, \beta).$$

Define the compounded random variable

$$X = X_1 + X_2.$$

Then the cumulative distribution function (CDF) of X is given by

$$F_X(x) = P(X \leq x) = \sum_{t=0}^x P(X = t),$$

where

$$P(X = t) = \sum_{k=0}^t P_{LB}(k) P_{GG}(t-k).$$

Therefore,

$$F_X(x) = \sum_{t=0}^x \sum_{k=0}^t P_{LB}(k) P_{GG}(t-k).$$

Substituting the expressions of the component PMFs, the CDF can be written as

$$F_X(x) = \sum_{t=0}^x \sum_{k=0}^t \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} I_{k,t},$$

where

$$I_{k,t} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+t-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

Proof: By definition, the cumulative distribution function of a discrete random variable X is

$$F_X(x) = P(X \leq x) = \sum_{t=0}^x P(X = t).$$

Since $X = X_1 + X_2$ and X_1 and X_2 are independent random variables, the probability mass function of X is obtained by the convolution formula

$$P(X = t) = \sum_{k=0}^t P(X_1 = k) P(X_2 = t-k).$$

Let $P_{LB}(k)$ denote the PMF of the Lindley-Binomial distribution and $P_{GG}(y)$ denote the PMF of the Gamma-Geometric distribution. Substituting these expressions into the convolution formula gives

$$P(X = t) = \sum_{k=0}^t P_{LB}(k) P_{GG}(t-k).$$

Therefore

$$F_X(x) = \sum_{t=0}^x \sum_{k=0}^t P_{LB}(k) P_{GG}(t-k).$$

Using the explicit forms of $P_{LB}(k)$ and $P_{GG}(y)$ yields

$$F_X(x) = \sum_{t=0}^x \sum_{k=0}^t \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} I_{k,t},$$

where

$$I_{k,t} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+t-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

Hence the required cumulative distribution function of the proposed compounded distribution is obtained.

Theorem 1.5 : Moment Generating Function

Let X_1 and X_2 be two independent random variables such that

$$X_1 \sim LB(n, \lambda), \quad X_2 \sim GG(\alpha, \beta).$$

Define the compounded random variable

$$X = X_1 + X_2.$$

Then the moment generating function (MGF) of X is given by

$$M_X(t) = E(e^{t(X_1+X_2)}).$$

$$M_X(t) = E(e^{tX}) = M_{X_1}(t)M_{X_2}(t),$$

where $M_{X_1}(t)$ and $M_{X_2}(t)$ denote the MGFs of the Lindley-Binomial and Gamma-Geometric distributions respectively.

Further,

$$M_{X_1}(t) = \sum_{k=0}^n e^{tk} P_{LB}(k),$$

and

$$M_{X_2}(t) = \sum_{y=0}^{\infty} e^{ty} P_{GG}(y).$$

Therefore the MGF of the proposed compounded distribution becomes

$$M_X(t) = \left(\sum_{k=0}^n e^{tk} P_{LB}(k) \right) \left(\sum_{y=0}^{\infty} e^{ty} P_{GG}(y) \right).$$

Substituting the explicit expressions of the component PMFs yields

$$M_X(t) = \sum_{k=0}^n \sum_{y=0}^{\infty} e^{t(k+y)} \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} J_{k,y},$$

where

$$J_{k,y} = \int_0^\infty \theta^{k+\alpha} (1-\theta)^{n+y-k} (1+\theta) e^{-(\lambda+\beta)\theta} d\theta.$$

Proof : The moment generating function of a random variable X is defined as

$$M_X(t) = E(e^{tX}).$$

Since the proposed random variable is defined as

$$X = X_1 + X_2,$$

we obtain

Because X_1 and X_2 are independent random variables, the expectation can be written as

$$M_X(t) = E(e^{tX_1})E(e^{tX_2}).$$

Thus

$$M_X(t) = M_{X_1}(t)M_{X_2}(t).$$

The MGF of X_1 is

$$M_{X_1}(t) = \sum_{k=0}^n e^{tk} P_{LB}(k),$$

and the MGF of X_2 is

$$M_{X_2}(t) = \sum_{y=0}^{\infty} e^{ty} P_{GG}(y).$$

Substituting the expressions of the PMFs of the Lindley-Binomial and Gamma-Geometric distributions gives

$$M_X(t) = \sum_{k=0}^n \sum_{y=0}^{\infty} e^{t(k+y)} \binom{n}{k} \frac{\lambda^2}{1+\lambda} \frac{\beta^\alpha}{\Gamma(\alpha)} J_{k,y}.$$

Hence the moment generating function of the proposed compounded distribution is obtained.

Theorem 1.6 : Mean and Variance

Let X be the compounded random variable defined as

$$X = X_1 + X_2$$

where

$$X_1 \sim LB(n, \lambda), \quad X_2 \sim GG(\alpha, \beta),$$

and X_1 and X_2 are independent random variables.

Then the mean and variance of X are given by

$$E(X) = E(X_1) + E(X_2),$$

and

$$Var(X) = Var(X_1) + Var(X_2).$$

Further,

$$E(X_1) = \sum_{k=0}^n k P_{LB}(k),$$

$$E(X_2) = \sum_{y=0}^{\infty} y P_{GG}(y),$$

and

$$Var(X_1) = E(X_1^2) - [E(X_1)]^2,$$

$$Var(X_2) = E(X_2^2) - [E(X_2)]^2.$$

Proof : The expectation of a random variable X is defined as

$$E(X) = E(X_1 + X_2).$$

Using the linearity property of expectation we obtain

$$E(X) = E(X_1) + E(X_2).$$

Similarly, the variance of X is

$$Var(X) = Var(X_1 + X_2).$$

Since X_1 and X_2 are independent random variables,

$$Var(X) = Var(X_1) + Var(X_2).$$

The mean of X_1 and X_2 can be obtained from their respective probability mass functions as

$$E(X_1) = \sum_{k=0}^n k P_{LB}(k)$$

and

$$E(X_2) = \sum_{y=0}^{\infty} y P_{GG}(y).$$

Similarly, their variances are obtained from the second moments.

Hence the mean and variance of the proposed compounded distribution follow directly.

Theorem 1.7 : Survival Function and Hazard Rate

Let X be the compounded random variable defined by

$$X = X_1 + X_2$$

where $X_1 \sim LB(n, \lambda)$ and $X_2 \sim GG(\alpha, \beta)$.

Then the survival function of X is given by

$$S(x) = P(X > x) = 1 - F_X(x),$$

where

$$F_X(x) = \sum_{t=0}^x P(X = t).$$

The hazard rate function of X is defined as

$$h(x) = \frac{P(X = x)}{P(X \geq x)}.$$

Substituting the PMF of the proposed distribution gives

$$h(x) = \frac{\sum_{k=0}^x P_{LB}(k) P_{GG}(x - k)}{1 - \sum_{t=0}^{x-1} \sum_{k=0}^t P_{LB}(k) P_{GG}(t - k)}.$$

Proof : The survival function of a random variable X is defined as

$$S(x) = P(X > x).$$

For a discrete distribution this can be written as

$$S(x) = 1 - F_X(x).$$

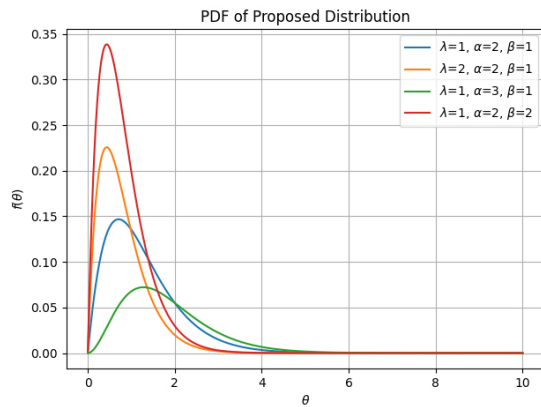


Fig. 1. Probability density function (PDF) of the proposed distribution for different parameter values $(\lambda, \alpha, \beta) = (1, 2, 1), (2, 2, 1), (1, 3, 1), (1, 2, 2)$. The curves illustrate the flexibility of the model in capturing various shapes.

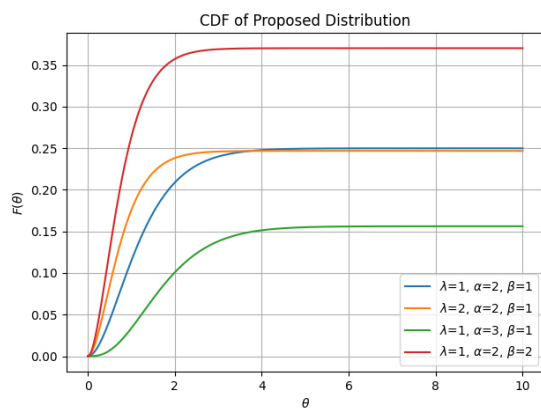


Fig. 2. Cumulative distribution function (CDF) of the proposed distribution for different parameter values $(\lambda, \alpha, \beta) = (1, 2, 1), (2, 2, 1), (1, 3, 1), (1, 2, 2)$. The figure shows the increasing behavior of the distribution function.

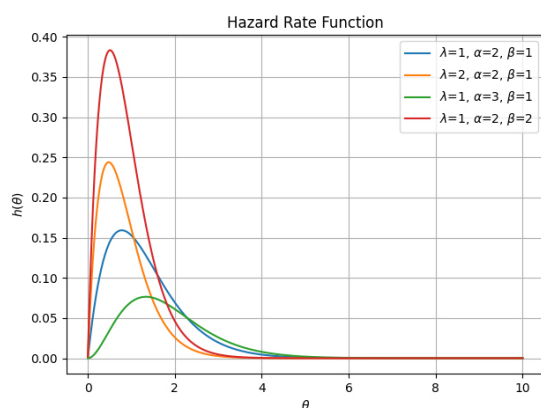


Fig. 3. Hazard rate function (HRF) of the proposed distribution for different parameter values $(\lambda, \alpha, \beta) = (1, 2, 1), (2, 2, 1), (1, 3, 1), (1, 2, 2)$. The model exhibits unimodal hazard rate shapes, demonstrating its applicability in reliability and lifetime data modeling.

Since

$$F_X(x) = P(X \leq x) = \sum_{t=0}^x P(X = t),$$

we obtain

$$S(x) = 1 - \sum_{t=0}^x P(X = t).$$

Using the convolution form of the probability mass function

$$P(X = t) = \sum_{k=0}^t P_{LB}(k)P_{GG}(t - k),$$

the survival function follows.

The hazard rate function for a discrete distribution is defined as

$$h(x) = \frac{P(X = x)}{P(X \geq x)}.$$

Substituting the PMF of the proposed distribution yields the required hazard rate expression.

The shapes of the PDF, CDF, and hazard rate function of the proposed distribution for various parameter values. These plots indicate that the proposed model is flexible enough to accommodate different shapes of density and hazard functions, which makes it useful in reliability and lifetime data analysis.

2. PARAMETER ESTIMATION

Let $X_1, X_2, \dots, X_n$ be a random sample drawn from the proposed distribution with probability mass function $P(X = x; \theta)$, where $\theta = (\lambda, \alpha, \beta)$ is the vector of unknown parameters.

2.1 Maximum Likelihood Estimation

The likelihood function is given by

$$L(\theta) = \prod_{i=1}^n P(X = x_i; \theta). \quad (1)$$

Taking logarithm, the log-likelihood function becomes

$$\ell(\theta) = \sum_{i=1}^n \log P(X = x_i; \theta). \quad (2)$$

The maximum likelihood estimates are obtained by solving the score equations

$$\frac{\partial \ell(\theta)}{\partial \lambda} = 0, \quad (3)$$

$$\frac{\partial \ell(\theta)}{\partial \alpha} = 0, \quad (4)$$

$$\frac{\partial \ell(\theta)}{\partial \beta} = 0. \quad (5)$$

The solutions of these equations give the MLEs $\hat{\theta} = (\hat{\lambda}, \hat{\alpha}, \hat{\beta})$.

2.2 Method of Moments

Let the first two sample moments be

$$m_1 = \frac{1}{n} \sum_{i=1}^n x_i, \quad (6)$$

$$m_2 = \frac{1}{n} \sum_{i=1}^n x_i^2. \quad (7)$$

Let the corresponding theoretical moments be

$$E(X) = \mu_1(\theta), \quad (8)$$

$$E(X^2) = \mu_2(\theta). \quad (9)$$

The moment estimators are obtained by solving

$$m_1 = \mu_1(\theta), \quad m_2 = \mu_2(\theta). \quad (10)$$

2.3 Least Squares Estimation

Let the empirical cumulative distribution function be

$$F_n(x_i) = \frac{i}{n+1}, \quad i = 1, 2, \dots, n. \quad (11)$$

The least squares estimator minimizes the function

$$S(\theta) = \sum_{i=1}^n [F(x_i; \theta) - F_n(x_i)]^2. \quad (12)$$

Thus the estimator is given by

$$\hat{\theta} = \arg \min_{\theta} S(\theta). \quad (13)$$

2.4 Cramér-von Mises Estimation

The Cramér-von Mises estimation method minimizes

$$C(\theta) = \frac{1}{12n} + \sum_{i=1}^n \left[F(x_i; \theta) - \frac{2i-1}{2n} \right]^2. \quad (14)$$

The estimator is

$$\hat{\theta} = \arg \min_{\theta} C(\theta). \quad (15)$$

2.5 Anderson-Darling Estimation

The Anderson-Darling objective function is

$$A(\theta) = -n - \frac{1}{n} \sum_{i=1}^n (2i-1) [\ln F(x_i; \theta) + \ln (1 - F(x_{n-i+1}; \theta))]. \quad (16)$$

The estimator is obtained by

$$\hat{\theta} = \arg \min_{\theta} A(\theta). \quad (17)$$

2.6 Fisher Information Matrix

The Fisher information matrix is defined as

$$I(\theta) = \begin{pmatrix} I_{\lambda\lambda} & I_{\lambda\alpha} & I_{\lambda\beta} \\ I_{\alpha\lambda} & I_{\alpha\alpha} & I_{\alpha\beta} \\ I_{\beta\lambda} & I_{\beta\alpha} & I_{\beta\beta} \end{pmatrix}, \quad (18)$$

where

$$I_{ij} = -E \left[\frac{\partial^2 \ell(\theta)}{\partial \theta_i \partial \theta_j} \right]. \quad (19)$$

Table 1. Monte Carlo simulation results for bias and mean squared error of parameter estimators

Method	Bias _{λ}	Bias _{α}	Bias _{β}	MSE _{λ}	MSE _{α}	MSE _{β}
MLE	-0.6272	-1.1272	-0.1272	0.4172	1.2944	0.0400
MOM	1.1184	8.9574	-0.4624	1.4649	4810.6597	0.2683
LSE	-1.3834	-1.8834	-0.8834	1.9140	3.5474	0.7806
CVM	-1.3828	-1.8828	-0.8828	1.9124	3.5452	0.7796
AD	-1.3804	-1.8804	-0.8804	1.9057	3.5360	0.7753

Here $\ell(\theta)$ is the log-likelihood function given by

$$\ell(\theta) = \sum_{i=1}^n \log P(X = x_i; \theta). \quad (20)$$

The asymptotic variance of the maximum likelihood estimator is

$$\text{Var}(\hat{\theta}) \approx I^{-1}(\theta), \quad (21)$$

where $I^{-1}(\theta)$ is the inverse of the Fisher information matrix.

3. SIMULATION RESULTS

The results of the Monte Carlo simulation study are summarized in Table 1. The table presents the bias and mean squared error (MSE) values of the parameter estimators corresponding to the parameters λ , α , and β for the proposed distribution.

From the simulation results, it can be observed that the maximum likelihood estimator (MLE) performs reasonably well compared to the other estimators, producing relatively smaller bias and mean squared error values for most parameters. In particular, the MSE value associated with the parameter β is quite small, indicating good estimation accuracy.

The method of moments (MOM) estimator exhibits relatively large bias and extremely large mean squared error for the parameter α , which indicates that the MOM estimator is less reliable for estimating the parameters of the proposed distribution under the considered simulation setting.

The least squares estimator (LSE), Cramér–von Mises estimator (CVM), and Anderson–Darling estimator (AD) produce similar results with moderate bias and MSE values. Among these methods, the Anderson–Darling estimator provides slightly smaller mean squared error values compared to the LSE and CVM estimators.

Overall, the simulation results indicate that the maximum likelihood estimator provides relatively better performance among the considered methods, while the AD, CVM, and LSE estimators also

provide stable estimates. However, the method of moments appears to be less efficient for estimating the parameters of the proposed distribution.

7. CONCLUSION

In this study, a new compounded distribution obtained by combining the Lindley–Binomial and Gamma–Geometric distributions has been proposed. The proposed model provides a flexible framework for modeling count data and reliability-related phenomena. Several important statistical properties of the proposed distribution were derived, including the probability mass function, cumulative distribution function, and moment generating function. In addition, important statistical measures such as the mean and variance were obtained. Reliability characteristics such as the survival function and hazard rate function were also investigated.

Different parameter estimation techniques were considered for estimating the unknown parameters of the proposed distribution. These methods include maximum likelihood estimation (MLE), method of moments (MOM), least squares estimation (LSE), Cramér–von Mises estimation (CVM), and Anderson–Darling estimation (AD). A Monte Carlo simulation study was conducted to evaluate the performance of these estimators in terms of bias and mean squared error.

The simulation results indicate that the maximum likelihood estimator (MLE) performs relatively well compared with the other estimation methods, producing smaller bias and mean squared error values for most parameters. The LSE, CVM, and AD estimators also provide stable estimates with moderate bias and MSE values. On the other hand, the method of moments (MOM) estimator shows larger bias and extremely large mean squared error for the parameter α , indicating that it is less efficient for estimating the parameters of the proposed distribution under the considered simulation setting.

Overall, the proposed distribution demonstrates considerable flexibility and can serve as a useful alternative model for analyzing count data and reliability phenomena. The theoretical developments

and simulation results confirm the potential applicability of the model in various fields such as reliability analysis, actuarial science, and applied probability.

DECLARATIONS

Ethical Approval

We affirm that this manuscript is an original work, has not been previously published, and is not currently under consideration for publication in any other journal or conference proceedings. All authors have reviewed and approved the manuscript, and the order of authorship has been mutually agreed upon.

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Conflicts of Interest

The authors declare that they have no financial or personal interests that could have influenced the research and findings presented in this paper. The authors alone are responsible for the content and writing of this article.

Authors' contributions

All authors contributed equally in the preparation of this manuscript.

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