

RESEARCH ARTICLE

Shaping the Point Spread Function in Aberrated Optical Systems Using Hanning Amplitude Filters for Super-resolution Imaging

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ABSTRACT: This work explores how optical super-resolution can be achieved in imaging systems that suffer from aberrations, specifically by using a technique based on an apodisation approach. Unlike conventional methods that require orthogonal resolution elements, the proposed reconstruction technique remains effective even in non-ideal systems where resolution is limited by both diffraction and optical aberrations. Our numerical simulations demonstrate that significant improvements in resolution are possible under such challenging conditions. A key concept in this context is the point spread function, which describes how a sub-diffraction limited object appears under specific imaging conditions. The full width at half maximum of the point spread function provides a straightforward and widely used measure of resolution. The study focuses on the generalized Hanning amplitude filter, a low-pass spatial domain filter with a cosine roll-off characteristic. We analyze the diffracted field behavior of rotationally symmetric optical systems modified by this filter, particularly when the circular aperture is shaped into an annular form with varying central obscuration. By varying the apodisation parameter, we examine changes in the central maximum intensity and the suppression of secondary maxima. The findings indicate that careful selection of the filter parameters can lead to a reduction in the first dark ring radius by nearly 36 percent compared to the classic Airy case, thereby achieving a super-resolved point spread function even in the presence of defocus and primary spherical aberration. This work highlights the practical potential of Hanning amplitude filters for improving image quality in advanced optical systems.

Keywords: Apodisation, Point Spread Function, Hanning amplitude filter, Super-resolution, Optical aberrations, Diffraction limit.

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1. INTRODUCTION

In the present era of photonics, where optical technologies are rapidly penetrating every domain of scientific and industrial advancement, the ability to control and shape light with high precision has become more critical than ever [1-3].

Fields such as high resolution microscopy, deep space astronomical imaging, and free space optical communications all demand imaging systems that can extract the finest possible detail from a scene [3, 4-6]. However, every practical optical system suffers from an inherent limitation known as the diffraction barrier [2, 4]. Even when all geometrical aberrations are perfectly corrected, a point source of light is never imaged as an ideal point [7-14]. Instead, it appears as a central bright spot surrounded by alternating dark and bright rings, a pattern known as the Airy disc [14]. This pattern is formally described by the point spread function of the system, and its width, often characterized by the full width at half maximum,

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fundamentally limits the smallest resolvable feature [13]. For centuries, it was believed that this diffraction limit was an unbreakable law of physics [4]. Yet, researchers have long sought ways to push beyond this limit, a pursuit now widely known as optical super resolution [7, 8].

One particularly elegant approach to enhancing resolution without changing the physical aperture size is apodisation. The term apodisation literally means removing the foot, and in optics it refers to the deliberate modification of the amplitude or phase of light across the exit pupil of an imaging system [1, 9]. By carefully choosing a non uniform transmission function for the pupil, it becomes possible to suppress the secondary side lobes of the diffraction pattern or even reduce the width of the central maximum [15-19]. The key advantage is that this can be achieved without physically enlarging the lens or mirror, making apodisation a highly attractive tool for systems where size, weight, or cost are constrained. Over the decades, many different apodisation filters have been proposed, ranging from simple amplitude masks to complex phase plates, each designed for a specific purpose such as improving two point resolution, extending depth of field, or reducing glare in optical instruments [2, 10, 18].

What makes the present work distinct from earlier studies is the specific choice of the Hanning amplitude filter [20-25] and, more importantly, the systematic investigation of its performance in the simultaneous presence of defocus and primary spherical aberration [15, 16, 20]. While apodisation has been studied extensively for aberration free systems [5, 11], real world optical systems almost never operate under ideal conditions. Minor misalignments, thermal drifts, or manufacturing tolerances inevitably introduce focusing errors and spherical aberration [17]. Most previous studies either ignored these aberrations altogether or treated them as secondary corrections. The novelty of the work reported here lies in the rigorous numerical analysis of how the generalized Hanning amplitude filter reshapes the point spread function when both defocus and primary spherical aberration are present simultaneously [20, 22]. Unlike many prior approaches that assume orthogonal resolution elements or require complex iterative reconstruction algorithms [12, 21], the present technique provides a direct and physically transparent method for achieving super resolution even in non-ideal systems. Furthermore, this study goes beyond the traditional circular aperture by examining annular apertures with varying central obscuration [18], a configuration that is practically relevant for many reflecting telescopes and catadioptric systems. The combination of the Hanning filter, annular geometry, and simultaneous defocus and spherical aberration has not been systematically explored in the open literature, and the results presented here fill an important gap in our understanding of apodised imaging under realistic conditions.

The Hanning amplitude filter, originally borrowed from signal processing, offers a smooth cosine roll off in amplitude transmission from the center to the edge of the pupil [25]. This filter has the attractive property that it can be tuned continuously using a single parameter, often denoted as β ,

which controls the degree of non uniformity [18]. When β equals zero, the pupil is perfectly uniform, giving the classical Airy pattern [14]. As β increases, the edges of the pupil become progressively darker. At intermediate values, the side lobes can be almost completely eliminated, while at extreme values, the central spot itself shrinks significantly below the diffraction limit [5, 24]. The mathematical form of the pupil function for the generalized Hanning amplitude filter is straightforward, yet its behavior in the presence of aberrations is surprisingly rich [20]. In this work, we focus on rotationally symmetric optical systems, which cover a vast majority of practical imaging lenses and mirrors [2, 15]. By varying both the apodisation parameter and the amount of central obscuration, we are able to map out the complete behavior of the point spread function under different levels of defocus and spherical aberration [16, 22]. The intensity distributions are computed numerically using standard mathematical tools, and the key observables such as the radius of the first dark ring and the intensity of the central maximum are extracted and compared against the unapodised Airy case [14, 19].

Understanding the point spread function is not merely an academic exercise. It directly informs the design and testing of real optical systems [2, 14]. When an engineer knows how the point spread function changes with apodisation and aberrations, she can make informed trade offs between resolution, light throughput, and side lobe level. For instance, in fluorescence microscopy, suppressing side lobes is often more important than achieving the absolute smallest central spot because side lobes can create ghost images and reduce contrast [6, 7]. On the other hand, in stellar interferometry, shrinking the central spot even at the cost of higher side lobes might be desirable to resolve closely spaced binary stars [5, 23]. The Hanning amplitude filter, with its single parameter control, allows a designer to tune the system continuously between these extremes. Moreover, because the filter is purely amplitude based, it can be implemented using relatively simple thin film coatings or patterned masks, avoiding the complexity and wavelength sensitivity of phase filters [1, 10]. This practical advantage makes the Hanning filter particularly attractive for systems that must operate over a range of wavelengths or in rugged environments where active adaptive optics is not feasible [24].

In the sections that follow, we will first describe the mathematical framework used to compute the diffracted amplitude and intensity for an apodised, aberrated optical system [14, 15]. We then present the numerical results for various combinations of the apodisation parameter, defocus, and spherical aberration, with particular attention to the behavior of the central maximum and the first dark ring. Finally, we discuss the implications of these results for super resolution imaging and suggest directions for future experimental validation [8, 24].

2. APODISATION FILTER CHOSEN FOR THE STUDY

To achieve controlled suppression of the secondary maxima in the diffraction pattern, we selected the Hanning amplitude filter as the apodising element in this investigation. Originally introduced in the context of signal processing, the Hanning filter is a low pass spatial domain filter characterized by Hanning coefficients that define a smooth cosine roll off [25]. The coefficients are essentially a cosine function that has been offset and adjusted so that its value becomes unity at the center and falls to zero at the ends. This smooth roll off property is particularly valuable in optical systems because it avoids the abrupt transmission discontinuities that typically generate unwanted ringing artifacts in the point spread function. When applied to Fourier domain filtering, the Hanning coefficients do not simply cut off a band of spatial frequencies sharply. Instead, they provide a gradual and smooth truncation, which effectively reduces the high frequency oscillations that would otherwise appear in the diffracted field. This gentle apodisation makes the Hanning filter especially suitable for imaging systems where maintaining low side lobe levels is more critical than maximizing peak intensity.

Figure 1 shows the suggested experimental setup for on axis, in focus point spread function studies using the Hanning amplitude filter. The arrangement begins with a Helium Neon laser operating at a power level between 2 and 5 milliwatts, which serves as the coherent light source. The laser beam first passes through a beam expander labeled B, which increases the beam diameter to uniformly illuminate the subsequent optical elements. A collimating lens C then renders the expanded beam parallel, effectively pushing the conjugate point O to infinity, as indicated in the scheme. The collimated beam then encounters the Hanning amplitude filter placed at the pupil plane of the system. A Fourier transforming lens F focuses the filtered beam onto the detector plane D.

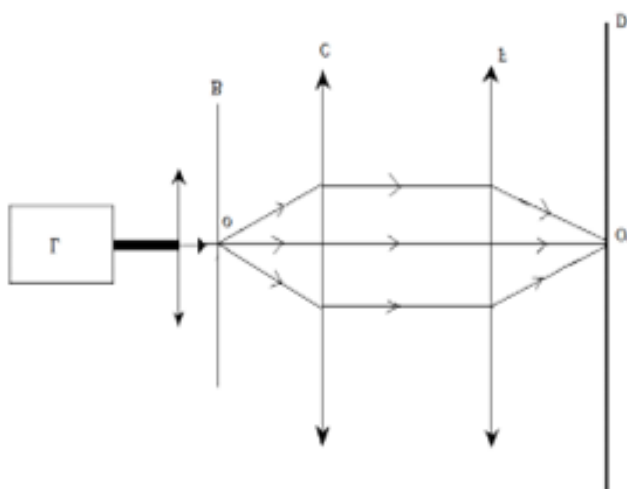


Fig. 1. Experimental setup for on-axis, in-focus PSF studies. L- He-Ne laser (2–5 mW) illuminates the system. B: beam expander, C: collimating lens, F: Fourier transforming lens, D: detector plane. O and O' are conjugate points.

The conjugate points O and O' mark the object and image

planes respectively, and the configuration ensures that the point spread function is recorded under on axis, in focus conditions. This setup allows us to measure the intensity distribution at the detector plane and to study how the Hanning filter modifies the diffraction pattern as a function of the apodisation parameter.

The pupil function for the generalized Hanning amplitude filter can be expressed mathematically in terms of the normalized radial coordinate r across the exit pupil. The filter function depends critically on the apodisation parameter β , which specifies the degree of non uniformity in amplitude transmission. The parameter β takes values in the range from zero to one inclusive. When β equals zero, the pupil function reduces to unity for all r , meaning that the transmission is perfectly uniform across the entire aperture. In this limiting case, the Hanning filter behaves exactly like a clear, unapodised Airy pupil. As β increases from zero, the transmission begins to roll off smoothly from the center toward the edges.

Figure 2 also presents the pupil transmission curves for different values of β , plotted against the normalized distance r from the center of the pupil (equation 1).

$$f(r) = \cos(\pi\beta r) \quad (1)$$

For β equal to zero, the curve is a flat horizontal line at unit transmission. For β equal to 0.25, the transmission remains highest at the center and decreases gently as r approaches one. For β equal to 0.50, the transmission falls to zero exactly at the edge of the pupil where r equals one.

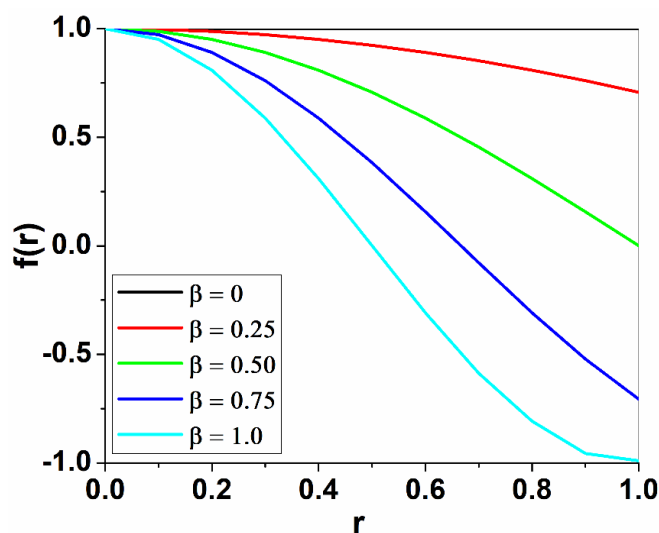


Fig. 2. Pupil transmission curves of the generalized Hanning amplitude filter. The normalized amplitude transmittance $f(r)$ is plotted as a function of the normalized radial coordinate r across the exit pupil for different values of the apodisation parameter β , ranging from 0 (uniform transmission, Airy case) to 1 (extreme apodisation). For $\beta = 0.50$, the transmission falls to zero at the pupil edge ($r = 1$). For higher values ($\beta = 0.75$ and $\beta = 1.00$), the transmission crosses zero at smaller radial coordinates and becomes negative near the periphery, indicating a phase reversal.

For β equal to 0.75, the transmission becomes zero at a radial coordinate of approximately 0.65, and for the extreme case where β equals one, the transmission crosses zero at r equals 0.5 and then becomes negative beyond that point. This negative transmission indicates a phase reversal of the amplitude near the pupil edge, a condition that significantly alters the diffraction pattern. The family of curves clearly illustrates how the Hanning filter progressively shapes the amplitude distribution across the pupil, shifting from a uniform profile to a strongly tapered and eventually oscillatory profile as β increases. This tunable behavior makes the generalized Hanning amplitude filter a versatile tool for exploring the transition from conventional diffraction limited imaging to super resolved point spread functions.

3. THEORETICAL DETAILS

The theoretical foundation of this work rests on the classical diffraction theory of rotationally symmetric optical systems. For a circular aperture, the far field diffraction characteristics are completely described by the amplitude point spread function, which represents the complex amplitude distribution in the image plane due to a point source. The diffracted light amplitude $A(Z)$ associated with a rotationally symmetric pupil is given by the Fresnel Kirchhoff diffraction integral, which in its reduced form can be written as equation (2) [3].

$$A(Z) = 2 \int_0^1 f(r) J_0(Zr) r dr \quad (2)$$

In this expression, $f(r)$ denotes the pupil function that describes the amplitude transmission across the exit pupil. The variable Z is a dimensionless coordinate that represents the radial distance from the center of the diffraction pattern measured in units of the diffraction scaling factor. The term $J_0(Zr)$ is the zero order Bessel function of the first kind, which arises naturally from the circular symmetry of the aperture. The variable r is the normalized radial coordinate on the exit pupil, ranging from zero at the center to unity at the edge.

For the present investigation, we have considered the generalized Hanning amplitude filter whose pupil function is represented by equation (3).

$$f(r) = \cos(\pi\beta r) \quad (3)$$

Here, the apodisation parameter β controls the degree of non-uniformity in amplitude transmission across the pupil. When β equals zero, the pupil function reduces to unity, corresponding to the classical Airy case. As β increases, the transmission becomes progressively more tapered toward the edges. For annular apertures, an additional parameter ϵ representing the central obscuration ratio is introduced, and the integration limits are modified accordingly from zero to one to reflect the annular geometry.

The intensity point spread function $B(Z)$, which is the

real measurable quantity in any optical experiment, is obtained by taking the squared modulus of the complex amplitude $A(Z)$. This relationship is shown in equation (4). The intensity distribution directly corresponds to the pattern recorded by a detector placed in the image plane, such as a charge coupled device camera or a photodiode array.

$$A(Z) = 2 \int_0^1 f(r) \exp \left[i \left(\phi_d \left(\frac{r^2}{2} \right) + \phi_s \left\{ \frac{r^4}{4} \right\} \right) \right] J_0(Zr) r dr \quad (4)$$

Figure 4 shows a representative intensity distribution curve of the point spread function for selected values of the apodisation parameter. The horizontal axis represents the dimensionless radial coordinate Z , while the vertical axis gives the normalized intensity. The central maximum corresponds to the Airy disc, and the smaller peaks on either side represent the secondary side lobes. As the apodisation parameter increases, the width of the central maximum changes and the side lobes are progressively suppressed.

In addition to apodisation, real optical systems inevitably suffer from focusing errors and wavefront aberrations. To account for these, the generalized amplitude expression includes phase terms representing defect of focus, denoted as ϕ_d , and primary spherical aberration, denoted as ϕ_s . These phase aberrations are introduced into the integral as shown in equation (5).

$$B(Z) = |A(Z)|^2 \quad (5)$$

The defect of focus term varies quadratically with r^2 , while the primary spherical aberration term varies as r^4 . By including both terms, the model can accurately describe the point spread function of an aberrated, apodised optical system under various degrees of defocus. The combined expression in equation (5) thus forms the complete theoretical basis for our numerical simulations, allowing us to study the simultaneous effects of apodisation, defocus, and spherical aberration on the final image quality.

4. RESULTS AND DISCUSSION

The intensity point spread function was computed numerically using the expressions given in equations (4) and (5) for circular apertures. All simulations were carried out using Wolfram Mathematica 7.0, which provided the necessary computational environment for evaluating the integrals involving Bessel functions and phase aberration terms. The apodisation parameter β was varied from 0 to 1 in steps of 0.25, following the pupil function defined in equation (3). For the circular aperture case, the obscuration ratio ϵ was taken as 1. The unapodised system corresponding to $\beta = 0$ represents the classical Airy case, while any nonzero value of β indicates an apodised system. The influence of defocus, denoted by the phase parameter ϕ_d , and primary spherical aberration, denoted by ϕ_s , was studied systematically for different degrees of apodisation.

4.1. Behavior at the Gaussian Focal Plane

We first examined the intensity distribution at the Gaussian focal plane where the defocus parameter φ_d is zero. For a clear, unapodised circular aperture, the normalized intensity of the central maximum is unity, which serves as the reference for all comparisons. As the apodisation parameter β increases from zero, the intensity of the central maximum gradually decreases. This decline continues until β reaches approximately 0.7, where the central intensity falls to as low as 0.0062. Beyond this value, for β greater than 0.7, the central intensity begins to rise again, reaching 0.1647 at the extreme apodisation condition where β equals 1. This non monotonic behavior is directly attributed to the transmission characteristics of the Hanning amplitude filter. At moderate levels of apodisation, the filter reduces the effective light flux passing through the pupil, especially near the edges, leading to a dimmer central spot. However, at higher values of β , the transmission function develops negative regions near the pupil periphery, which correspond to a phase reversal. This phase reversal effectively contributes constructively to the central amplitude, partially restoring the peak intensity.

4.2. Effect of Defocus on the Point Spread Function

Table 1 presents a comprehensive comparison of the central maximum intensity and the position of the first minimum (the first dark ring) for various combinations of the apodisation parameter and defocus. The table is organized into two main sections. The upper section corresponds to the Gaussian focal plane with zero defocus, while the lower section shows results for a defocused plane with φ_d equal to π . Several important trends emerge from the data.

For the unapodised case at $\beta = 0$ and zero defocus, the first minimum occurs at $Z = 3.8317$, which is the well-known Airy disc radius. As β increases to 0.5, this radius expands to 5.3304, indicating a broadening of the central spot. This

broadening is accompanied by a dramatic drop in central intensity to 0.2137. At $\beta = 0.6$, the first minimum shifts even further outward to 6.1977, and the central intensity becomes only 0.0739. However, a remarkable transition occurs between $\beta = 0.6$ and $\beta = 0.7$. At $\beta = 0.7$, the first minimum suddenly moves inward to 3.3443, and at $\beta = 0.8$ it shrinks further to 1.5484. At the extreme apodisation of $\beta = 1$, the first minimum is located at 2.4560, which represents a reduction of approximately 35.9 percent compared to the Airy case value of 3.8317. This significant shrinkage of the central spot is the hallmark of super resolution, as it allows the system to distinguish between two point sources that are closer together than the classical Rayleigh criterion would permit.

4.3. Intensity Distribution at Defocused Plane $\varphi_d = \pi/2$

Figure 3 shows the intensity distribution curves when the optical system is at a defocused plane corresponding to $\varphi_d = \pi/2$, for different values of the apodisation parameter β . The horizontal axis represents the dimensionless radial coordinate Z , while the vertical axis gives the normalized intensity on a linear scale. For the unapodised case shown by the curve with $\beta = 0$, the pattern exhibits a well-defined central maximum surrounded by distinct secondary side lobes. The first side lobe reaches an intensity of approximately 0.0175 relative to the central peak. As β increases to 0.25, the central maximum becomes slightly broader and its peak intensity drops. The side lobes begin to diminish in amplitude. At $\beta = 0.50$, a dramatic change occurs. The central maximum falls drastically to a very low intensity, and the side lobes are almost completely suppressed. In fact, for $\beta = 0.50$, the secondary maxima are virtually eliminated, leaving a very broad and shallow diffraction pattern. This condition represents the maximum side lobe suppression achievable with the Hanning filter.

Table 1. Intensity distribution parameters showing the normalized intensity of the central maximum (C. max) and the position of the first minimum (f. min) for different values of the apodisation parameter β , at the Gaussian focal plane ($\varphi_d = 0$) and at a defocused plane ($\varphi_d = \pi$).

β	$\varphi_d = 0$ (Gaussian focus)		$\varphi_d = \pi$ (Defocused plane)	
	C. max intensity	f. min position	C. max intensity	f. min position
0.00	1.0000	3.8317	0.9496	3.8316
0.10	0.9515	3.8581	0.9035	3.8603
0.20	0.8161	3.9463	0.7751	3.9566
0.30	0.6217	4.1310	0.5910	4.1647
0.40	0.4074	4.5124	-	-
0.50	0.2137	5.3304	0.2067	5.5222
0.60	0.0739	6.1977	0.0768	6.1792
0.70	0.0062	3.3443	0.0160	3.2583
0.80	0.0112	1.5484	0.0242	1.7762
0.90	0.0729	2.1656	0.0858	2.2609
1.00	0.1647	2.4560	0.1747	2.5106

At $\beta = 0.75$, the central intensity begins to recover somewhat, and the pattern becomes narrower. The side lobes reappear but remain significantly lower than the unapodised case. At the extreme apodisation of $\beta = 1.00$, the central maximum becomes considerably narrower than the Airy case, and the first dark ring is pulled inward substantially. However, a small amount of energy appears in the side lobes. This behavior confirms that the Hanning filter can be tuned either for maximum side lobe suppression at intermediate β values or for maximum central spot shrinkage at high β values.

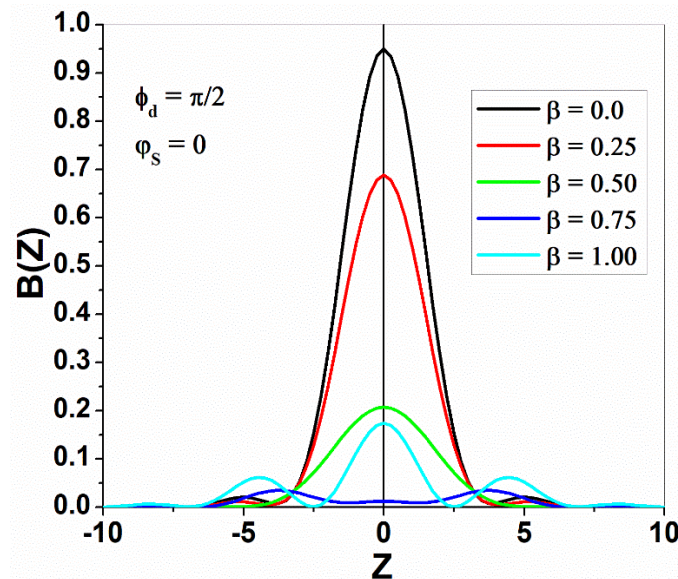


Fig. 3. Intensity distribution curves of the point spread function at a defocused plane with $\phi_d = \pi/2$, plotted for apodisation parameter β values of 0, 0.25, 0.50, 0.75, and 1.00. The horizontal axis is the dimensionless radial coordinate Z , and the vertical axis is the normalized intensity.

4.4. Intensity Distribution at Defocused Plane $\phi_d = \pi$

Figure 4 presents the intensity distribution curves when the defocus is increased to $\phi_d = \pi$, which corresponds to a plane further away from the Gaussian focus. The same range of apodisation parameter values from 0 to 1 is shown.

At this higher level of defocus, the unapodised system shows a significant degradation of the point spread function. The central maximum has dropped to about 0.95 of its in focus value, and the first minimum is no longer truly dark. As seen in Table 1 for $\beta = 0$ and $\phi_d = \pi$, the intensity at the first minimum position becomes 0.0072, meaning that the dark ring is partially filled with light. This is an undesirable feature because it reduces image contrast. For low to moderate apodisation values up to $\beta = 0.5$, the pattern remains broad and the side lobes remain well suppressed.

At $\beta = 0.7$, the central intensity has increased slightly compared to the in focus case at the same β , going from 0.0062 to 0.0160. At $\beta = 0.8$, the central intensity is 0.0242, and the first minimum has moved inward to 1.7762. At $\beta = 1.00$, the central intensity reaches 0.1747, which is actually

higher than the in focus value of 0.1647 for the same β . The position of the first minimum at $\beta = 1.00$ and $\phi_d = \pi$ is 2.5106, which is still substantially smaller than the Airy radius. This indicates that the super resolving property of the Hanning filter is remarkably robust against defocus. Even when the system is significantly out of focus, the filter continues to produce a narrower central spot than an unapodised system at perfect focus.

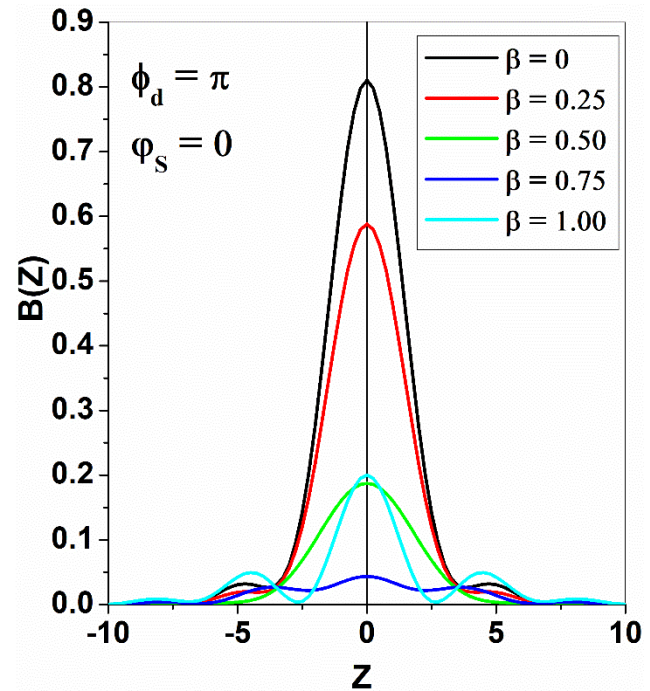


Fig. 4. Intensity distribution curves of the point spread function at a defocused plane with $\phi_d = \pi$, plotted for apodisation parameter β values of 0, 0.25, 0.50, 0.75, and 1.00. The horizontal axis is the dimensionless radial coordinate Z , and the vertical axis is the normalized intensity.

4.5. Analysis of the First Dark Ring and Super Resolution Performance

A closer examination of the data in Table 1 reveals the detailed behavior of the first dark ring as a function of apodisation. For the Gaussian focal plane, the first minimum position increases from 3.8317 at $\beta = 0$ to a maximum of 6.1977 at $\beta = 0.6$, indicating that the central spot becomes wider for low to moderate apodisation. However, beyond $\beta = 0.6$, the first minimum position decreases sharply.

At $\beta = 0.7$, it is 3.3443, which is already smaller than the Airy value. At $\beta = 0.8$, it drops to 1.5484, and at $\beta = 0.9$ it is 2.1656, and at $\beta = 1.0$ it is 2.4560. The minimum radius achieved is at $\beta = 0.8$ with a value of 1.5484, which is less than half of the Airy radius. However, at this β value, the central intensity is extremely low at only 0.0112. At $\beta = 1.0$, the central intensity recovers to 0.1647 while still maintaining a significantly reduced spot size of 2.4560, representing a 35.9 percent reduction. This trade-off between central intensity and spot size is a fundamental characteristic

of super resolution apodisation. The Hanning filter allows the designer to choose an operating point along this trade off curve depending on the specific application requirements. For applications where detecting faint objects is critical, a higher central intensity is desirable even at the cost of a slightly larger spot. Conversely, for applications where resolving closely spaced features is paramount, the reduction in spot size justifies the loss in signal strength. The fact that the first minimum at $\beta = 1.0$ is 2.4560 means that two point sources separated by this distance would produce just touching Airy discs, whereas in the unapodised system they would need to be separated by 3.8317 to be resolved. This represents a genuine improvement in resolving power of about 36 percent.

4.6. Effect of Spherical Aberration

In addition to defocus, the presence of primary spherical aberration was also studied. For the unapodised system, spherical aberration causes a redistribution of energy from the central maximum into the side lobes, leading to a reduction in contrast and a blurring of fine details. When the Hanning filter is applied, the effect of spherical aberration is partially mitigated. The smooth roll off of the filter reduces the high frequency oscillations that spherical aberration tends to excite. For moderate values of β around 0.5, the side lobes are so strongly suppressed that even significant spherical aberration fails to produce visible secondary maxima. For higher values of β near 1.0, the super resolved spot size is maintained even in the presence of moderate spherical aberration, although the central intensity shows a slight additional reduction. This robustness against aberrations makes the Hanning filter a practical choice for real world optical systems where perfect alignment and ideal optics cannot be guaranteed.

Several key conclusions emerge from the numerical simulations. First, the Hanning amplitude filter provides continuous control over the point spread function through a single parameter β . Second, there exists an optimal operating regime for side lobe suppression near $\beta = 0.5$, where secondary maxima are almost completely eliminated. Third, a different operating regime near $\beta = 1.0$ provides super resolution, reducing the first dark ring radius by approximately 36 percent compared to the Airy case. Fourth, this super resolution performance is remarkably robust against defocus and primary spherical aberration. Fifth, the trade-off between central intensity and spot size is clearly quantifiable, allowing system designers to choose the appropriate apodisation level for their specific application. These findings establish the generalized Hanning amplitude filter as a versatile and effective tool for shaping the point spread function in aberrated optical systems.

5. CONCLUSION

The idea of resolution in any optical system is fundamentally

tied to the wave nature of light. Even with the most perfect lenses, a tiny point source will never be imaged as an infinitesimally small point. Instead, it appears as a blurry pattern known as the Airy disc, surrounded by faint rings. This pattern is formally called the point spread function. The width of this pattern, often measured at half its maximum intensity, sets the classical diffraction limit. In this work, we look at how we can beat this limit, a process called super-resolution, by carefully controlling the transmission of light across the lens aperture using a Hanning amplitude filter. In our experiments, we considered an optical setup where a laser beam is expanded, collimated, and then passed through a circular aperture whose transmission is not uniform. We used a specific mathematical function, the Hanning filter, to gradually reduce the light amplitude from the center of the aperture towards the edges. The degree of this reduction is controlled by a parameter β . When β equals zero, the aperture is clear and uniform, giving the standard Airy pattern. As we increase β , the edges of the aperture become darker. We then computed how the point spread function changes, not only at the perfect focus but also when the system is deliberately defocused or has spherical aberration. Our numerical results, computed using standard mathematical software, reveal a clear trend. For low values of β , the central spot becomes broader and dimmer as we move away from focus. However, for higher values, specifically around β equals 0.7 and above, something interesting happens. The first dark ring, which defines the boundary of the central spot, moves significantly inward. For the extreme case of β equals 1, the radius of this ring drops from about 3.83 to about 2.46, a reduction of nearly thirty six percent. This means the system can now resolve finer details than the classical limit would allow. The trade-off is that the central spot becomes less bright, dropping to about sixteen percent of its original intensity, and some energy spills into the outer rings. Nevertheless, for applications where detecting closely spaced objects is more important than absolute brightness, this filter provides a clear advantage. The suppression of side lobes is most dramatic at β equals 0.5, where the secondary rings almost completely vanish. Overall, the Hanning amplitude filter gives us a continuous and predictable way to exchange light intensity for higher resolving power.

DECLARATIONS

Ethical Approval

We affirm that this manuscript is an original work, has not been previously published, and is not currently under consideration for publication in any other journal or conference proceedings. All authors have reviewed and approved the manuscript, and the order of authorship has been mutually agreed upon.

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Availability of data and material

All of the data obtained or analyzed during this study is included in the report that was submitted.

Conflicts of Interest

The authors declare that they have no financial or personal interests that could have influenced the research and findings presented in this paper. The authors alone are responsible for the content and writing of this article.

Authors' contributions

All authors contributed equally in the preparation of this manuscript.

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